Modeling exchange rate, inflation, and interest rate in a small open economy

A data-based approach to estimating a New Keynesian model with rational expectations

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Objectives and findings

- **Novel Data-Driven Method:**
  The theoretical model is taken to the data with a strictly frequentist and full-information approach.
- **Avoiding Traditional Techniques:**
  This stands in contrast to the (Bayesian) approach where 'estimation' relies on a combination of calibration and Bayesian inference techniques.
- **Model Validation:**
  Aim at investigating the possibility of validating the theoretical model without having to impose its validity to a large extent in advance.

Objectives and findings

- **SVECM Appoach:**
  Utilize a SVECM approach where theory informs the long-term cointegrating structure, while its empirical quantification and the model's dynamic specification, are determined by the data.
- **Testing NKPC:**
  Enable a test for the empirical validity of the New-Keynesian Phillips Curve (NKPC) **in a fully structural setting.**
- **Results of Combining Frameworks:**
  Demonstrate that it may be possible to combine a theoretical New Keynesian RE framework with a frequentist estimation method.

Objectives and findings

- **Phillips Curve:**
  By avoiding a simultaneity bias in design and estimation, we have been able to identify a dynamic Phillips Curve.
- **Theoretical Framework Corroboration:**
  The central banks of both countries are also found to engage in symmetric and flexible inflation targeting.
- **Empirical Model Deviations:**
  Our empirical model deviates from a theoretical New-Keynesian rational expectation model in that the exchange rate equation is a backward-looking process
- **Alignment with Empirical Studies:**
  However, this persistence aligns with findings from other empirical studies (Benedictow and Hammersland (2023)).

Literature

- A substantial body of literature focuses on theoretical New Keynesian models, e.g Gali (2018).
- A substantial literature on empirical econometric time-series modeling, e.g. Hendry (1995).
- Limited research combining both theoretical and empirical models due to differing evaluation criteria.
- Achieving a balance between theoretical rigor and empirical fit presents significant challenges.
  - Calvo pricing has been used to incorporate lags to account for a sluggish response in the data using NK models(Calvo (1983))
  - A common empirical strategy is to trade off some fit and statistical properties in order to maintain theoretical validity.

Literature

- Validating the rational expectations hypothesis has turned out to be challenging, especially on Norwegian data.
- Norwegian studies find mixed support for the traditional NKPC theory (Bårdsen et al., 2004; Boug et al., 2017).
- Boug et al., 2017 reveal simultaneity and endogeneity issues. They also find that the real exchange rate may have a significant bearing on the process driving inflation.
- Empirical investigations of the NKPC in open economies have been conducted in various international studies(Leith and Malley (2007), Batini et al. (2005), and Bussière et al. (2014)).
- Common feature: demonstrate that movements in real exchange rates can significantly influence inflation dynamics

Literature

- Given that Norway is characterized as a small open economy, it should be particularly relevant to integrate the real exchange rate into the NKPC utilizing Norwegian data.
- However, few studies have explored this using a fully structural understanding of the data-generating process (DGP) with Norwegian data.
- This paper seeks to fill this gap by offering a comprehensive New Keynesian model framework for open economies, providing a basis for simultaneous modeling of the processes driving exchange rates, prices, and interest rates.

Theory: New-Keynesian model set-up of a two-country two-markets economy

- Aggregate relationships are derived from the underlying decisions of households and firms, each country producing a range of products being produced by country specific monopolists.
- The model is based on the optimizing behavior of infinitely-lived representative households and monopolistic firms under uncertainty and with rational expectations.
- Firms are monopolistic producers and produce imperfect substitutes and when they are allowed to change their prices (price rigidity) these are set so that the discounted present value of their total future expected profit is maximized.
- Symmetrical model with equal preferences and technologies (no home bias in consumption).

New-Keynesian model of a two-country two-markets economy continued

- To be more specific the New-Keynesian model is based on deviation from UIP in terms of a risk premium

$$i_t - i_t^* = E_t(q_{t+1}) - q_t + E_t(\pi_{t+1} - \pi_{t+1}^*) - \lambda_t$$

- Monetary policy is determined at home and abroad by two similar reaction functions (flexible inflation targeting with interest rate smoothing)

$$i_t - i_t^* = \sigma(\pi_t - \pi_t^*) + \rho(y_t - y_t^*) + \alpha(i_{t-1} - i_{t-1}^*)$$

- Companies set prices to maximize the expected discounted value of all future profits

$$\pi_t - \pi_t^* = \delta q_t + \beta E_t(\pi_{t+1} - \pi_{t+1}^*)$$

New-Keynesian model of a two-country two-markets economy continued

- The simultaneous structure

$$\underbrace{\begin{bmatrix} 1 & -\delta & 0 \\ 0 & 1 & 0 \\ \sigma & 0 & \alpha \end{bmatrix}}_{A} \underbrace{\begin{pmatrix} d\pi_t \\ q_t \\ dI_{t-1} \end{pmatrix}}_{z_t} = \underbrace{\begin{bmatrix} \beta & 0 & 0 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}}_{B} \mathrm{E}_t \underbrace{\begin{pmatrix} d\pi_{t+1} \\ q_{t+1} \\ dI_t \end{pmatrix}}_{z_{t+1}}$$

$$+ \underbrace{\begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & -\rho \end{bmatrix}}_{C} \underbrace{\begin{pmatrix} \lambda_t \\ dy_t \end{pmatrix}}_{x_t}, \text{ Multiplying by: } \begin{bmatrix} 1 & -\delta & 0 \\ 0 & 1 & 0 \\ \sigma & 0 & \alpha \end{bmatrix}^{-1} =>$$

New-Keynesian model of a two-country two-markets economy continued

$$\underbrace{\begin{pmatrix} d\pi_t \\ q_t \\ dI_{t-1} \end{pmatrix}}_{z_t} = \underbrace{\begin{bmatrix} \beta+\delta & \delta & -\delta \\ 1 & 1 & -1 \\ \sigma(\beta-\delta)/\alpha & -\sigma\delta/\alpha & (\sigma\delta+1)/\alpha \end{bmatrix}}_{A^{-1}B} \mathrm{E}_t \underbrace{\begin{pmatrix} d\pi_{t+1} \\ q_{t+1} \\ dI_t \end{pmatrix}}_{z_{t+1}}$$

$$+ \underbrace{\begin{bmatrix} \delta & 0 \\ 1 & 0 \\ \sigma\delta/\alpha & -\rho/\alpha \end{bmatrix}}_{A^{-1}C} \underbrace{\begin{pmatrix} \lambda_t \\ dy_t \end{pmatrix}}_{x_t}$$

Empirical data

- p and p^* $\Rightarrow$dp=p-p*: Core CPI, Norway and EA and their difference
  $\Rightarrow \pi = \Delta p$ and $\pi^* = \Delta p^*$ $\Rightarrow \Delta dp$ = π -π^* = $d\pi$
- I and I^* $\Rightarrow$dI=I-I* : Nominal interest rates, Norway and EA, and their difference
- s: Nok krone euro exchange rate $\Rightarrow$ q=s+p^*-p
- dy: Output gap difference: Norway and EA
- p^{oil}: Oil price (Brent Blend) in USD

Risk premium (λ) captured by the following variables:

- v: export value of oil and gas as a share of the total value of exports
- *FDI*: difference between inward foreign direct investments as a percentage of GDP in the euro area and in Norway.
- a: petroleum-related equity index
- *VIX*: volatility index related to the US SP500 stock market

The GUM: A Structural Vector Equilibrium Correction Model (SVECM)

$$
\underbrace{\begin{pmatrix} \Delta d\pi_t \\ \Delta q_t \\ \Delta I_t \end{pmatrix}}_{\Delta z_t} = \underbrace{\begin{pmatrix} c_\pi \\ c_q \\ c_I \end{pmatrix}}_{c} + \underbrace{\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\phi_q & -\phi_q\alpha_q \\ 0 & 0 & -\phi_I \end{bmatrix}}_{F} \underbrace{\begin{pmatrix} d\pi_{t-1} \\ q_{t-1} \\ dI_{t-1} \end{pmatrix}}_{z_{t-1}}
$$

$$
+ \underbrace{\begin{bmatrix} 0 & \delta & 0 \\ \phi_q\omega_q & 0 & 0 \\ \phi_I\omega_I & 0 & 0 \end{bmatrix}}_{H} \underbrace{\begin{pmatrix} d\pi_t \\ q_t \\ dI_t \end{pmatrix}}_{z_t} + \underbrace{\begin{bmatrix} \beta_\pi & 0 & 0 \\ 0 & \beta_q & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{J} \mathrm{E}_t \underbrace{\begin{pmatrix} d\pi_{t+1} \\ q_{t+1} \\ dI_{t+1} \end{pmatrix}}_{z_{t+1}}
$$

$$
+ \underbrace{\begin{bmatrix} 0 & 0 \\ -\phi_q & 0 \\ 0 & \phi_I\rho_I \end{bmatrix}}_{K} \underbrace{\begin{pmatrix} \lambda_{t-1} \\ dy_t \end{pmatrix}}_{x_t} + \sum_{i=0}^{k} \Pi \, \Delta v_{t-i} + \Theta d_t + \underbrace{\begin{pmatrix} \epsilon_t^\pi \\ \epsilon_t^q \\ \epsilon_t^I \end{pmatrix}}_{\epsilon_t},
$$

The parsimounious SVECM representation

$$d\pi_t \equiv \Delta dp_t = \underset{(6.19)}{-25.91} + \underset{(0.029)}{0.12}\, q_t + \underset{(0.18)}{0.95}\widehat{\Delta dp_{t+1}} + D_{\pi,t} + \tilde{\epsilon}_t^{\pi},$$

$$\Delta q_t = \underset{(0.06)}{0.20} + \underset{(0.10)}{0.11}\Delta q_{t-1} + \underset{(0.087)}{0.35}\,\Delta q_{t-2} - \underset{(0.017)}{0.082}\Delta p_t^{oil} + \underset{(0.016)}{0.075}\Delta p_{t-1}^{oil}$$
$$-\underset{(0.009)}{0.045}\Delta dI_t + \underset{(0.008)}{0.03}\,\Delta dI_{t-1} + \underset{(0.0015)}{0.004}\,\Delta(FDI \cdot D_{17})_{t-1}) + \underset{(0.05)}{0.13}\Delta(a \cdot D_{17})_{t-1}$$
$$-\underset{(0.03)}{0.10}(q_{t-1} + 0.023\, a_{t-1} - 0.004\, FDI_{t-1} + 0.23\, v_{t-1} - 0.004\, VIX_{t-1})$$
$$-\underset{(0.001)}{0.004}(dI_{t-1} - d\pi_t) + D_{q,t} + \underset{(0.0003)}{0.0004}\,\Delta VIX_t + \tilde{\epsilon}_t^{q},$$

$$\Delta I_t = \underset{(0.058)}{0.012} + \underset{(0.059)}{0.25}\,\Delta I_{t-1} + \underset{(0.06)}{0.90}\Delta I_t^* - \underset{(0.08)}{0.19}\Delta I_{t-1}^*$$
$$-\underset{(0.014)}{0.036}(dI_{t-1} - (dp_t - dp_{t-4})) + \underset{(0.006)}{0.015} dy_t + D_{I,t} + \epsilon_t^{i}.$$

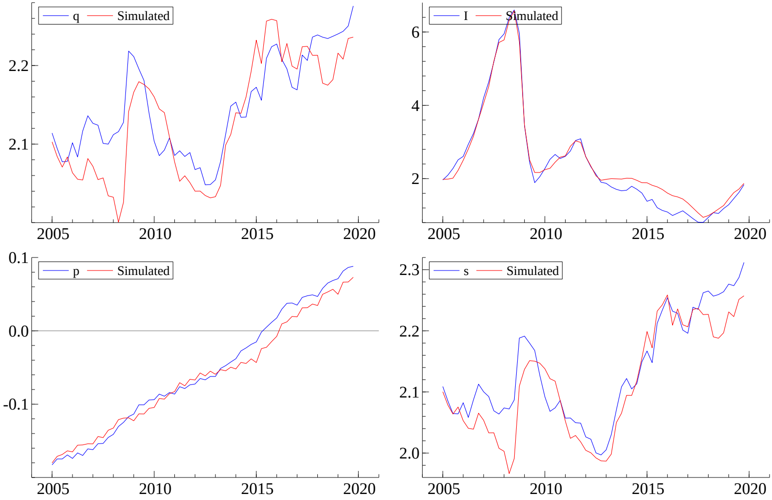
Long run equilbrium solution

$$s_t = p_t - p_t^* \equiv dp_t$$

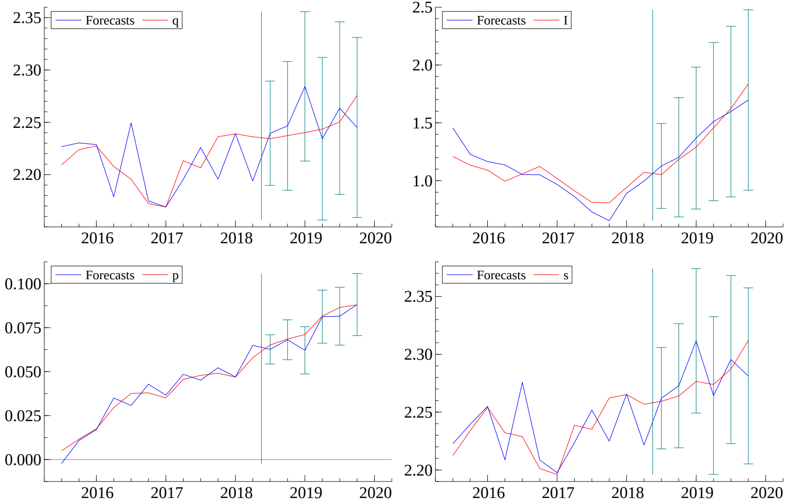
$$q_t = -0.023\,a_t + 0.004\,FDI_t - 0.23\,v_t + 0.004\,VIX_t - 0.036\,(dI_t - \Delta dp_{t+1})$$

$$dI_t = (dp_{t+1} - dp_{t-3}) + 0.44\,dy_{t+1}$$

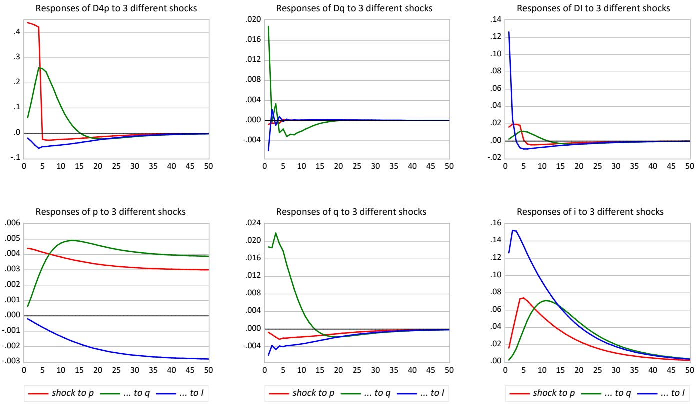
Dynamic Simulations



Dynamic Forecasts



Impulse responses



Summary and conclusion

- Suggest a (novel) data-driven method for how to take a New-Keynesian rational expectation model to the data utilizing a purely frequentist approach.
- Demonstrate that it is possible to estimate a New Keynesian rational expectation model without having to resort to calibration or Bayesian inference techniques.
- Our study provides empirical support for a model of the simultaneous process driving bilateral exchange rates, relative prices and interest rates, that is mostly in accordance with a New Keynesian model of a two country two markets economy.
- Contrary to New Keynesian theory framework, our empirical model indicates a backward-looking exchange rate process.

Summary and conclusion

- This persistence is on the other hand consistent with findings from other empirical studies and does also suggest the presence of multiple equilibria (Sunspots).
- Results corroborate the theoretical framework anticipated by a two-country-two-market model, in that the central banks of both countries are found to engage in symmetric and flexible inflation targeting.
- An unexpected yet significant finding of our study is the validation of the Phillips curve.
- This finding is robust to boot and suggests potential simultaneity biases in earlier research on Norwegian data.

THANK YOU!

Stationarity and Cointegration tests

Variable	Lagged level parameter	T-value	Conclusion
s_t	-0.043(2)	-1.088	I(1)
p_t	0.006(2)	1.06	I(1)
p_t^*	-0.016(2)	-2.02	I(1)
v_t	-0.09(2)	-1.31	I(1)
op_t	-0.075(2)	-2.07	I(1)
s_t + p_t^*- p_t	-0.07(1)	-1.6	I(1)
a_t	-0.055(2)	-1.74	I(1)
FDI_t	-0.049(2)	-1.12	I(1)
I_t	-0.067(2)	-2.83	I(1)
I_t^*	-0.036(2)	-1.70	I(1)
dI_t	-0.099(2)	-3.36**	I(0)
$d\pi_t$	-0.43(2)	-3.18**	I(0)
$dI_t - d\pi_{t+1}$	-0.39(2)	-3.05**	I(0)
$dp_t - dp_{t-4}$	-0.12(2)	-3.00**	I(0)
$dI_t - (dp_{t+1} - dp_{t-3})$	-0.14(2)	-3.12**	I(0)
$\tilde{q}_t$	-0.195(2)	-3.15**	I(0)
y_t	-0.18(3)	-2.9**	I(0)
$q_t + 0.023\ a_t + 0.227\ v_t\ - 0.004\ \{FDI + VIX\}_t$			I(0)

The parsimounious SVECM: Identities

$$Cla_t = \Delta Cla_t + Cla_{t-1}$$
$$Clb_t = \Delta Clb_t + Clb_{t-1}$$
$$Clc_t = \Delta Clc_t + Clc_{t-1}$$
$$q_t = \Delta q_t + q_{t-1}$$
$$p_t = \Delta p_t + p_{t-1}$$
$$I_t = \Delta I_t + I_{t-1}$$
$$s_t = q_t + p_t - p*_t$$

,

IV-estimation of the Phillips Curve relationship

$$\Delta dp_t = \underset{(9.20)}{-27.38} + \underset{(0.044)}{0.13}\, q_t + \underset{(0.202)}{0.997} \Delta dp_{t+1} + \hat{\tilde{D}}_{\pi,t} + \epsilon_t^{\pi},$$

AR 1-5 test: $F(5, 69) = 1.8872$ [0.1078]
ARCH 1-4 test: $F(4, 72) = 0.99259$ [0.4172]
Normality test: $\chi^2(2) = 2.5899$ [0.2739]
Hetero test: $F(6, 72) = 1.2598$ [0.2868]
Hetero-X test: $F(7, 71) = 1.5023$ [0.1805]
RESET23 test: $F(2, 72) = 0.82029$ [0.4444]

Sargent's specification test: $\chi^2(6) = 11.599$ [0.072]

Instrumented variables: Δdp_{t+1} and q_t

Instruments:

$$\{const, D_{\pi,t}, \Delta p_t^{oil}, \Delta p_{t-1}^{oil}, \Delta y_t^*, \Delta y_{t-1}^*, \Delta I_t^*, \Delta I_{t-1}^*, dI_{t-1}, \Delta dp_{t-2}\}. \quad (3)$$